

Modified gravity with R -matter couplings and (non-)geodesic motion

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Abstract. We consider alternative theories of gravity with a direct coupling between matter and the Ricci scalar. We study the relation between these theories and ordinary scalar-tensor gravity, or scalar-tensor theories which include non-standard couplings between the scalar and matter. We then analyze the motion of matter in such theories, its implications for the Equivalence Principle, and the recent claim that they can alleviate the dark matter problem in galaxies.

1. Introduction

An explanation for the nature of the unknown form of energy, usually dubbed dark energy, which appears to constitute 76% of the energy budget of the universe [1] and is responsible for its late time accelerated expansion is still pending. The dominant role of gravity in the dynamics of the late time universe and the unexpected characteristics of dark energy, namely the fact that it violates the Strong Energy Condition and may even be as exotic as phantom energy, have lead some to the idea that dark energy might be nothing but an indication of the inability of General Relativity (GR) to describe gravity consistently at cosmological scales [2]. According to this approach, some other theory of gravity, which actually constitutes an extension of GR, might correctly describe the cosmological dynamics without dark energy and, at the same time, reduce to the usual GR phenomenology at scales where the latter has already been very successful.

Several theories have been put forth, often as toy models with the purpose of providing proofs of principle that such an explanation to the recent puzzles of cosmology can indeed be feasible. Most of them also seem to draw some motivation from high energy physics, as they are presented as candidates for a low energy effective action of some more fundamental theory (*e.g.*, String Theory). A typical example is that of the so-called $f(R)$ gravity [3]. These theories, which are proposed as minimal extensions of GR, are described by the action

$$S_f = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} f(R) + L_m(g_{ab}, \psi) \right], \quad (1)$$

where g is the determinant of the metric g_{ab} , $\kappa = 8\pi G$, $f(R)$ is a general function of the Ricci scalar R , L_m is the matter action, and ψ collectively denotes the matter

fields (we adopt the notations and conventions of Ref. [4]). When $f(R)$ is taken to be linear, this action reduces to the usual Einstein-Hilbert action. Then, Einstein's equations can be derived with standard metric variation, but also with an independent variation of the metric and of the connection [3, 4] called Palatini variation, under the crucial assumption that the matter action does not depend on the, now independent, connection [5]. Nevertheless, when $f(R)$ is not linear in R , the two variational principles lead to different theories: metric and Palatini $f(R)$ gravity \ddagger respectively (see [3, 6, 7, 8, 9] for early works and reviews). Additionally, one could decide to allow the independent connection to enter the matter action when the Palatini variation is used (which appears to be in accordance with the geometrical meaning of a connection to define the covariant derivative [5]), in which case the outcome is yet another distinct version of $f(R)$ gravity: metric-affine $f(R)$ gravity [10].

Much attention has been paid to $f(R)$ theories of gravity lately, especially concerning the cosmological evolution but also the various viability constraints [11] (with metric-affine $f(R)$ gravity being probably the exception). An extension of metric $f(R)$ gravity has been recently proposed in Ref. [12], in which the Ricci scalar R acquires an explicit coupling to the matter Lagrangian. The action of the theory is

$$S = \int d^4x \sqrt{-g} \left\{ \frac{f_1(R)}{2} + [1 + \lambda f_2(R)] L_m \right\}, \quad (2)$$

where L_m is the matter Lagrangian, $f_{1,2}$ are (*a priori* arbitrary) functions of the Ricci curvature R , and the usual coefficient of R in the Einstein-Hilbert action ($8\pi G$, where G is Newton's constant) has been absorbed in $f_1(R)$.

This action, and variants of it, were considered for various purposes: in Ref. [12], (2) was studied as an alternative to dark matter because it is claimed to give rise to phenomenology similar to MOND gravity [13] on galactic scales; in [14, 7], a variation of (2) was studied as an alternative to dark energy by setting $f_1(R) = R$ and keeping only the nonminimal coupling of matter to the Ricci curvature, in order to explain the current acceleration of the universe. Along the same line of thought, the idea of making the kinetic term of a (minimally coupled) scalar field dependent on the curvature, while keeping $f_1(R) = R$ was exploited in attempts to cure the cosmological constant problem [15, 16] (see also [17, 18]). In light of its relation with a scalar-tensor theory explained below, the action (2) is reminiscent also of the one describing a nonminimally coupled electromagnetic field [19].

Variation of the action (2) with respect to the metric yields

$$\begin{aligned} & F_1(R)R_{ab} - \frac{1}{2}f_1(R)g_{ab} - \nabla_a \nabla_b F_1(R) + g_{ab} \square F_1(R) \\ &= -2\lambda F_2(R)L_m R_{ab} + 2\lambda(\nabla_a \nabla_b - g_{ab} \square)L_m F_2(R) \\ &+ [1 + \lambda f_2(R)] T_{ab}, \end{aligned} \quad (3)$$

where $F_{1,2}(R) \equiv f'_{1,2}(R)$, the prime denotes differentiation with respect to the argument, and

$$T_{ab} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ab}}. \quad (4)$$

\ddagger In the Palatini formalism, the connection Γ^a_{bc} is independent of the metric. Therefore, R is replaced in the action (1) by $\mathcal{R} \equiv g^{ab}\mathcal{R}_{ab}$, where \mathcal{R}_{ab} is the Ricci tensor constructed with this independent connection.

As a consequence of the explicit coupling of the matter Lagrangian density to the Ricci curvature, there is an energy exchange between matter and gravity beyond the usual one always present in curved spaces, which is reflected in the non-vanishing of the covariant divergence of the matter stress-energy tensor T_{ab} . The corrected conservation equation assumes the form [12, 14, 7]

$$\nabla^b T_{ab} = \frac{\lambda F_2}{1 + \lambda f_2} (L_m g_{ab} - T_{ab}) \nabla^b R. \quad (5)$$

The fact that the stress energy tensor is manifestly not divergence-free (in the representation used to write down the action (2)) can be interpreted as a violation of the so-called metric postulates [20]. This seems to imply that the theory exhibits violations of the Equivalence Principle (EP). This, as well as the fact that one could potentially tune the parameter λ to reduce the effects of such violation below current experimental accuracy, has already been mentioned in Ref. [12]. Nevertheless, to the best of our knowledge, no detailed study of this feature of the theories described by (2) has been carried out so far. Even though the metric postulates can be a good criterion for constructing theories which do not violate the EP, one can not safely conclude that any theory that does not satisfy the metric postulates in some particular representation indeed violates the EP. First of all, the metric postulates are representation-dependent statements, *i.e.*, not invariant under field redefinitions [21]. In general, there might be other representations of the theory in which the metric postulates are indeed satisfied §. More importantly, even in theories that indeed exhibit violations of the EP, such as those considered here, the metric postulates or the non-conservation of energy do not themselves provide quantitative estimates of the deviations from the EP.

The purpose of this paper is exactly to shed some light on these issues. We begin by examining, in section 2, the relation between the theories described by the action (2), ordinary scalar-tensor gravity, and scalar-tensor theories with an anomalous coupling between matter and the scalar field. It is shown that the former can not be cast into the form of any of the latter (as usually done in “ordinary” $f(R)$ gravity without the R -matter coupling). This not only implies that the theory under consideration can not be cast away as a known theory, but also that the R -matter coupling can not be eliminated with simple field redefinitions. We proceed, in section 3, to examine the effect of the R -matter coupling on the motion of particles and fields. It is shown that massless particle trajectories and field propagation are actually unaffected (at least at high frequencies). More remarkably, even the motion of massive objects (such as perfect fluids or test particles) seems to remain unaffected. This implies that detecting deviations from geodesic motion in the theories under investigation might be more difficult than expected. On the other hand, it also casts doubts on whether they can actually account for the phenomenology for which they were introduced in [12].

2. Is R -coupled modified gravity a déjà vu?

It is customary in metric (and also Palatini) $f(R)$ gravity to introduce auxiliary scalar fields, in order to re-write the action as a scalar-tensor theory [24]. Let us check what

§ A characteristic example is scalar-tensor gravity: the metric postulates are satisfied in the Jordan frame but not in the Einstein frame. However, there are no violations of the Einstein EP, since the latter is a characteristic of the theory, not of its representations [21] (see [22, 23] for a discussion of the Einstein EP in nonminimally coupled scalar field theory, which is a particular realization of scalar-tensor gravity). It is just that in one frame this is made manifest.

would happen if we followed the same procedure here: we begin by introducing two scalars Ψ and ϕ and considering the action

$$S' = \int d^4x \sqrt{-g} \left\{ \frac{f_1(\phi)}{2} + [1 + \lambda f_2(\phi)] L_m + \Psi(R - \phi) \right\}. \quad (6)$$

The variation of this action with respect to Ψ yields $R = \phi$ and one recovers the action (2). Now, varying (6) with respect to ϕ yields

$$\Psi = \frac{1}{2} f'_1 + \lambda f'_2 L_m. \quad (7)$$

Assuming that at least one of $f_{1,2}$ is non-linear in R ($f'_1 \neq 0$ or $f'_2 \neq 0$, or both) ^{||} and replacing eq. (7) back in eq. (6) one gets (see also [14, 7])

$$S' = \int d^4x \sqrt{-g} \left\{ \frac{f_1(\phi)}{2} + [1 + \lambda f_2(\phi)] L_m + \left(\frac{1}{2} f'_1 + \lambda f'_2 L_m \right) (R - \phi) \right\}. \quad (8)$$

This action is obviously dynamically equivalent to the action (2). In the special case $\lambda = 0$, in which (2) reduces to the action (1), and (8) reduces to

$$S' = \int d^4x \sqrt{-g} \left\{ \frac{f_1(\phi)}{2} + L_m + \frac{1}{2} f'_1 (R - \phi) \right\}, \quad (9)$$

or, with the simple field redefinition $\Phi \equiv f'_1(\phi)$ and the introduction of the potential $V(\Phi) = (\phi f'_1 - f_1)/2$, it can be rewritten as a Brans-Dicke theory with a potential and vanishing Brans-Dicke parameter ω_0 [24]:

$$S' = \int d^4x \sqrt{-g} \left\{ \frac{\Phi R}{2} - V(\Phi) + L_m \right\}. \quad (10)$$

However, when $\lambda \neq 0$, one can see directly from the action (8) that the coupling between matter and R does not disappear. Even if we perform the field redefinition mentioned earlier and introduce the functions $V(\Phi)$, $U(\Phi) \equiv 1 + \lambda(f_2 - \phi f'_2)$, and $X(\Phi) \equiv f'_2$, at best we can write

$$S' = \int d^4x \sqrt{-g} \left\{ \frac{\Phi R}{2} - V(\Phi) + U(\Phi) L_m + \lambda X(\Phi) R L_m \right\}. \quad (11)$$

This is clearly not an ordinary scalar-tensor theory: first, there exists an unusual coupling between the scalar and the matter in the third term. This is reminiscent of extensions of scalar-tensor gravity which include similar couplings, such as the theories considered by Damour and Polyakov [26]. However, the presence of the R -matter (-scalar) coupling in the last term distinguishes the action (11) even from these generalized scalar-tensor theories. Alternative ways to introduce a single scalar are also unable to alleviate the R -matter coupling (see the Appendix).

Also, one should not be misled to think that the R -matter coupling can be eliminated, judging from action (6). Even though in eq. (6) there is indeed no explicit coupling between R and the matter, there is a coupling between Ψ and R . Ψ in turn is just an auxiliary scalar which is algebraically related to the matter fields through eq. (7). Consequently, the R -matter coupling is just made implicit through

^{||} The particular case in which both functions are linear in R , $f'_1 = f'_2 = 0$, is burdened with serious viability issues [25] and will not be studied further.

the introduction of an auxiliary (non-dynamical) field. Note the difference with ϕ (or Φ) which does carry dynamics (it is algebraically related to R but not to the matter).

Therefore, we conclude the following:

- Introducing scalar fields helps in avoiding the presence of non-linear functions of R in the action (as in ordinary $f(R)$ gravity).
- However, the theories under scrutiny cannot be written into the form of a scalar-tensor theory with a minimal coupling to matter.[¶] *I.e.* the R -matter coupling cannot be eliminated, or replaced by some unusual couplings between matter and the scalar field (it is, therefore, not an artifact of some peculiar representation).
- One could “hide” the R -matter coupling by considering the theory as a multi-scalar tensor theory (starting from action (6) and possibly considering also field redefinitions for the two scalars). However, in this case special attention should be paid to the fact that one of these scalar does not actually carry dynamics. This, for instance, distinguishes this theory from the multi-scalar-tensor theories already considered in the literature [27].

Consequently, the theories described by the action (2) can not be cast into the form of a scalar- or multi-scalar-tensor gravity which has already been extensively studied in the literature and its phenomenology is well known: The presence of the R -matter coupling which is exactly the subject of interest here, is what distinguishes these theories and deserves further attention.

3. R -coupling, geodesics, and EP violations

As already mentioned, the explicit coupling to the Ricci curvature described by the action (2) can potentially lead to non-geodesic motion and violations of the EP, judging from the fact that the stress-energy tensor is not divergence-free, as expressed by eq. (5). Although this is pointed out in Ref. [12] with the caveat that such violations could be controlled by the value of the parameter λ , this issue has not yet been addressed in detail. In this section we perform a more thorough analysis. First, we consider massless particles and high frequency fields, and then we move on to perfect fluids.

3.1. Massless fields

In [12], it is stated that the explicit coupling of the matter Lagrangian to the Ricci curvature does not alter the *null* geodesic equation: however, this statement is not proved, and the reader might be left with the impression that it is motivated by the resemblance of the action (2) with the one of Einstein frame scalar-tensor theories, which we have shown to be fallacious. As a matter of fact, the authors of [12] are more interested in the correction to the worldlines of *massive* particles and the corresponding MOND-like phenomenology; however, this aspect of the theory should not be left unchecked.

[¶] By scalar-tensor theory we mean a theory with a single scalar field mediating gravity. For theories with more than one scalars we use the term multi-scalar-tensor theory.

3.1.1. Null dust: The equation for null geodesics can be derived in a straightforward manner from the conservation equation of a null dust fluid (see *e.g.*, Ref. [29]). Therefore, if the R -coupling were to induce any corrections to the null geodesic equation, these would show up in this derivation.

The stress-energy tensor of a null dust is

$$T_{ab} = \rho u_a u_b, \quad u_c u^c = 0, \quad (12)$$

where ρ is the fluid energy density and u^c is the four-velocity of massless fluid particles. Since the perfect fluid is an “averaged” and not an exact description for matter, it is more common in this case to work directly with the stress-energy tensor and avoid any reference to a Lagrangian. Unfortunately, in our case this is not possible since L_m enters explicitly in the conservation equations (5). Even though more than one choices for the Lagrangian have been used in the literature [30, 31], the most natural (and general) choice seems to be simply the pressure $L_m = P$ [30].⁺ The corrected conservation equation (5) then becomes

$$\rho u^b \nabla_b u_a + \rho u_a \nabla^b u_b + u_a u_b \nabla^b \rho = -\frac{\lambda F_2}{1 + \lambda f_2} \rho u_a u^b \nabla_b R. \quad (13)$$

Now this equation can be written in the form

$$u^b \nabla_b u_a = \Theta u_a, \quad (14)$$

where

$$\Theta = -\nabla^b u_b + \frac{1}{\rho} u_b \nabla^b \rho - \frac{\lambda F_2}{1 + \lambda f_2} u^b \nabla_b R. \quad (15)$$

Eq. (14) states that the four-velocity is parallel-transported along the path: this is the definition of a geodesic curve. The fact that this equation differs from the more familiar form $u^c \nabla_c u^a = 0$ is simply because the latter is affinely parametrized. Therefore, the explicit R -coupling does not change the equation of null geodesics.

3.1.2. Massless scalar field: Let us consider now a massless scalar field ϕ described by the Lagrangian density and energy-momentum tensor [32]

$$L_m = -\frac{1}{2} \nabla_c \phi \nabla^c \phi, \quad (16)$$

$$T_{ab} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi. \quad (17)$$

With a correction proportional to $g_{ab} L_m - T_{ab} = -\nabla_a \phi \nabla_b \phi$, the (non-)conservation equation (5) is

$$\begin{aligned} & (\nabla^b \nabla_a \phi) \nabla_b \phi + (\nabla_a \phi) \square \phi - \frac{1}{2} (\nabla_a \nabla^c \phi) (\nabla_c \phi) - \frac{1}{2} (\nabla^c \phi) \nabla_a \nabla_c \phi \\ &= -\frac{\lambda F_2}{1 + \lambda f_2} (\nabla_a \phi) (\nabla_b \phi) \nabla^b R. \end{aligned} \quad (18)$$

By projecting onto $\nabla^a \phi$, one obtains the corrected Klein-Gordon equation

$$\square \phi + \frac{\lambda F_2}{1 + \lambda f_2} (\nabla_b \phi) \nabla^b R = 0. \quad (19)$$

⁺ Note that the choice of the Lagrangian is particularly meaningful here: in GR these different choices would anyway lead to the same field equations, unlike here where the actual expression for L_m enters the field equation of the theory explicitly. Therefore, all results strongly depend on this choice.

By taking the high frequency limit

$$\phi(x^a) = \phi_0(x^a) e^{iS(x^c)}, \quad (20)$$

with the phase S a rapidly varying function of x^c and the amplitude $\phi_0(x^a)$ slowly varying, and neglecting the gradients and second derivatives of ϕ_0 , eq. (19) implies

$$-S_a S^a + i\nabla^c S_c = -\frac{\lambda F_2}{1 + \lambda f_2} iS_b \nabla^b R, \quad (21)$$

where $S_c \equiv \nabla_c S$ is the phase gradient and the tangent to the worldline of the scalar particle in the geometric optics approximation. This equation yields

$$S_a S^a = 0 \quad (22)$$

(i.e., the spacetime trajectory is null), and

$$\nabla^c S_c = -\frac{\lambda F_2}{1 + \lambda f_2} S_b \nabla^b R, \quad (23)$$

which expresses the fact that the “scalar photon” is not transversal unless $\lambda = 0$ or S^b is orthogonal to the gradient of R . By further applying the covariant derivative operator to eq. (22) and using the fact that $[\nabla_a, \nabla_b] S = 0$, one obtains

$$S^a \nabla_a S^b = 0. \quad (24)$$

Therefore, the worldlines of scalar particles in the high frequency approximation are *null geodesics*. This conclusion would not change if one were to consider instead a massless, non-minimally coupled, scalar field described by the Lagrangian density

$$L_m = -\frac{1}{2} \nabla_c \phi \nabla^c \phi - \frac{\xi}{2} R \phi^2, \quad (25)$$

where ξ is a dimensionless coupling constant. In fact, the corresponding Klein-Gordon equation

$$\square \phi - \xi R \phi = -\frac{\lambda F_2}{1 + \lambda f_2} (\nabla_b \phi) \nabla^b R \quad (26)$$

contains the extra term $-\xi R \phi$ that can be interpreted as a tidal effect on the scalar field which is important only for long wavelengths and disappears in the high frequency limit, yielding again the null geodesic equation.

3.1.3. Maxwell field: Let us now consider the Maxwell field with Lagrangian density and energy-momentum tensor

$$L_m = -\frac{1}{16\pi} F_{cd} F^{cd}, \quad (27)$$

$$T_{ab} = \frac{1}{4\pi} \left(F_{ac} F_b{}^c - \frac{1}{4} g_{ab} F_{cd} F^{cd} \right). \quad (28)$$

The corrected conservation equation (5) yields

$$\nabla^b \left(F_{ac} F_b{}^c - \frac{1}{4} g_{ab} F_{cd} F^{cd} \right) = -\frac{\lambda F_2}{1 + \lambda f_2} F_{ac} F_b{}^c \nabla^b R. \quad (29)$$

By taking now the high frequency limit

$$A_b = C_b e^{iS} \quad (30)$$

with the phase $S(x^c)$ a rapidly varying function and C_b a slowly varying vector amplitude, and neglecting the derivatives of the latter, one obtains (using $F_{ab} = \partial_a A_b - \partial_b A_a$),

$$2\alpha_{ab}S^b - S_a \left[S^2 C^2 - (S_c C^c)^2 \right] = 0, \quad (31)$$

$$\nabla^b \alpha_{ab} - \frac{1}{2} \nabla_a \left[S^2 C^2 - (S_c C^c)^2 \right] = - \frac{\lambda F_2}{1 + \lambda f_2} \alpha_{ab} \nabla^b R, \quad (32)$$

where $S_c \equiv \nabla_c S$, $C^2 \equiv C_c C^c$, $S^2 \equiv S_c S^c$, and

$$\alpha_{ab} = C^2 S_a S_b - (C^c S_c) (S_a C_b + S_b C_a) + S^2 C_a C_b. \quad (33)$$

Eqs. (31) and (33) yield $\left[C^2 S^2 - (C_c S^c)^2 \right] S_a = 0$ and

$$C^2 S^2 = (C_c S^c)^2 \quad (34)$$

and eqs. (31) and (32) simplify to

$$2\alpha_{ab}S^b = 0, \quad (35)$$

$$\nabla^b [(1 + \lambda f_2) \alpha_{ab}] = 0. \quad (36)$$

The only difference with respect to the standard Maxwell equations in curved space is the term in λ in eq. (36). It is straightforward to see that, in this equation,

$$|\nabla^b \alpha_{ab}| \approx \left| \frac{\alpha_{ab}}{\lambda_{em}} \right| \gg |\alpha_{ab} \nabla^b [1 + \lambda f_2(R)]| \approx \left| \frac{\alpha_{ab} \lambda f_2}{L} \right|, \quad (37)$$

where λ_{em} is the wavelength of electromagnetic waves and L is the radius of curvature of spacetime. In the high frequency limit $\frac{\lambda_{em}}{L} \ll 1$, the corrections to standard optics coming from the term $\lambda f_2(R)$ in the Lagrangian disappear and photons follow null geodesics and are transversal. In other words, the non-minimal coupling corrections to the Maxwell equations can only affect long wavelengths, comparable to the radius of curvature of spacetime. In the high frequency limit, photons are transverse and propagate along null geodesics.

3.2. Perfect fluid with constant equation of state

Let us now turn our attention to massive matter fields and, for simplicity, let us consider a perfect fluid composed of non-relativistic or relativistic particles with constant barotropic equation of state $P = w\rho$, where ρ and P are the energy density and pressure, respectively, described by the stress-energy tensor

$$T_{ab} = (w + 1) \rho u_a u_b + w p g_{ab} \quad (38)$$

and by the Lagrangian $L_m = P = w\rho$. The corrected conservation equation (5) yields

$$\begin{aligned} & (w + 1) u_a u^b \nabla_b \rho + (w + 1) \rho u^b \nabla_b u_a \\ & + (w + 1) u_a \nabla^b u_b + w \nabla_a \rho = - \frac{\lambda (w + 1) F_2}{1 + \lambda f_2} \rho u_a u_b \nabla^b R. \end{aligned} \quad (39)$$

Projecting onto the direction of the fluid four-velocity u^a , one obtains

$$\frac{D\rho}{D\tau} + (w + 1) \rho \nabla^b u_b = - \frac{\lambda (w + 1) F_2}{1 + \lambda f_2} \rho \frac{DR}{D\tau}, \quad (40)$$

where τ is the proper time along the timelike fluid curves and $D/D\tau \equiv u^c \nabla_c$. The correction to the conservation equation is best seen in the weak-field limit, in which $\tau \sim t$, $\frac{D\rho}{D\tau} \simeq \frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho$, and

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = - \frac{\lambda(w+1)F_2}{1+\lambda f_2} \rho \left(\frac{\partial R}{\partial t} + \vec{v} \cdot \vec{\nabla} R \right). \quad (41)$$

Clearly, the fluid energy is not conserved and the right hand side of eq. (41) acts as a source term describing the energy injected into the fluid per unit time and per unit volume. This correction disappears if $\lambda \rightarrow 0$, in vacuo, quantum vacuo ($w = -1$), or when $DR/D\tau = 0$.

We now project eq. (39) onto the direction normal to the four-velocity by the use of the projection operator h^a_c defined by $h_{ab} \equiv g_{ab} + u_a u_b$ (recall that, in our signature, $u^c u_c = -1$). This gives, after easy manipulations,

$$\frac{Du^a}{D\tau} \equiv \frac{du^a}{d\tau} + \Gamma^a_{bc} u^b u^c = f^a, \quad (42)$$

where

$$f^a = \frac{1}{\rho(1+w)} \left[\frac{\lambda F_2}{1+\lambda f_2} (L_m - P) \nabla_c R + \nabla_c P \right] h^{ac}. \quad (43)$$

The last term in square brackets, proportional to the pressure gradient, is the usual term that appears in GR and encapsulates the force exerted on a fluid element due to the fluid pressure (it is not to be attributed to the coupling between matter and R , nor does it signal any new effect). As mentioned earlier, a natural choice for the Lagrangian of a perfect fluid is $L_m = P$ [30]. Substitution into eq. (43) yields

$$f^a = \frac{1}{\rho(1+w)} (\nabla_c P) h^{ac}, \quad (44)$$

i.e., remarkably, the only force is due to the pressure gradient already present in GR. For instance, in the case of dust with $P = 0$, it is $f^c = 0$. Therefore, we conclude that there is no extra force which can be attributed to the coupling between the matter and R even for the case of a perfect fluid. The fact that energy is indeed not conserved for this fluid does not contradict this result. On the contrary, one can readily see from the right hand side of eq. (39) that the flow of energy only occurs along the direction of u^c , *i.e.*, aligned with the fluid worldlines. Only the time component of the force is non-zero, while its spatial components in the frame comoving with the fluid always vanish. This kind of force can have no effect on the motion because, due to the normalization $u^c u_c = -1$, the only meaningful component of the four-acceleration a^c and of the four-force $f^c \propto a^c$ is the one perpendicular to the four-velocity ($a_c u^c = 0$), *i.e.*, the spatial one. For a timelike four-velocity u^c , a four-acceleration parallel to u^c is necessarily zero, which is what we recover here. In a Newtonian analogy, the correction to the equations of motion would correspond to the introduction of a spatially homogeneous, but time-dependent, potential energy.

The fact that no extra force appears for a perfect fluid was missed in Ref. [12]. We will discuss the implications of this fact, to some extent, in section 3.4. For the moment, let us stress the following: In [12] the authors use the opposite signature than the one used here, but also in Refs. [30] where the result $L_m = P$ is derived. The use of a different signature is also the reason for the sign difference in the parenthesis and in front of P between eq. (43) and the corresponding equation of Ref. [12]. Both equations are consistent given the signature adopted. However, when replacing L_m in

this equation one has to take the signature into account as well: since the gravitational action changes sign after a signature change, the sign of the matter action has to be changed for consistency, *i.e.*, $L_m = -P$ with the signature convention of [12]. Note also that in a follow-up publication by Bertolami and Páramos studying the effects of the coupling discussed here on stellar configurations [33], it is assumed that $L_m = P$, even though the same signature as in Ref. [12] is used, which is the opposite of the one used here and in Refs. [30]. This certainly affects the results derived and, therefore, one has to be cautious about the conclusions stated there until the effect of this inconsistency in the analysis is assessed.

It is worth noting that the above conclusions apply to a massive scalar as well. This can be understood through the fact that a massive scalar admits a perfect fluid representation. Alternatively, it can be seen directly by the fact that the right hand side of eq. (5) is proportional to $L_m g_{ab} - T_{ab}$. Given the definition of the stress energy tensor, any part of the Lagrangian L_m which does not explicitly depend on the metric, such as the potential of a scalar field, does not contribute to this term. Massless or massive, a scalar field will always lead to an energy flow along its motion only, resulting into a vanishing extra force.

3.3. Conformal frames and energy conservation

We have already discussed the fact that the R -matter coupling can not be eliminated by a conformal transformation of the metric for a generic matter Lagrangian. This is evident in the fact that R couples in the same way to all the terms that a matter Lagrangian might be split into (kinetic, potential, *etc.*). However, one could think of employing the tool of conformal transformations in order to see how particular matter fields are affected by this coupling. More specifically, consider a matter field whose Lagrangian does indeed transform as $L_m \rightarrow AL_m$ under the transformation $g_{ab} \rightarrow \Omega^2 g_{ab}$, where A is some power of the scale factor Ω . Then it does seem possible, under certain conditions, to find a conformal frame in which energy is conserved for this field (and, consequently, particles associated with this field follow geodesics of this metric). Recall also that geodesic motion is a characteristic of the theory, not of its representations, so changing conformal frames in an effort to find one that makes it manifest is perfectly legitimate.* Indeed, we can re-derive some of the results of the previous section with the use of conformal frames. This, besides being a way of verifying their validity, is also a nice exercise for realizing the ambiguities of the metric postulates, already pointed out in [21].

We begin by recalling that, if under the conformal transformation

$$g_{ab} \rightarrow \tilde{g}_{ab} = \Omega^2 g_{ab} \quad (45)$$

the stress energy tensor T_{ab} transforms as

$$T^{ab} \rightarrow \tilde{T}^{ab} = \Omega^s T^{ab}, \quad (46)$$

where s is an appropriate conformal weight, then one can easily verify that the following equation relates the covariant divergencies of \tilde{T}_{ab} and T_{ab} [4]:

$$\tilde{\nabla}_a \tilde{T}^{ab} = \Omega^s \nabla_a T^{ab} + (s+6)\Omega^{s-1} T^{ab} \nabla_a \Omega - \Omega^{s-1} T \nabla^b \Omega, \quad (47)$$

* We are referring to whether a particle follows geodesics of some metric. The Einstein EP requires that the particles follow geodesics of some metric, not necessarily the one the action is written with. Of course, if particles corresponding to different fields follow geodesics of different metrics, then this signals a violation of the universality of free fall. Such a violation however, can only be detected in Eötvös-type experiments by comparing the trajectories of different particles and not by detecting deviation of a single species from geodesic motion (see section 3.4).

where $\tilde{\nabla}_c$ is the covariant derivative associated with \tilde{g}_{ab} and $T = g_{ab}T^{ab}$. Now, by imposing the condition that $\tilde{\nabla}_a \tilde{T}^{ab} = 0$, *i.e.*, that energy is conserved in the new frame, we get

$$\nabla_a T^{ab} = \frac{1}{\Omega} [Tg^{ab} - (s+6)T^{ab}] \nabla_b \Omega. \quad (48)$$

We rewrite here eq. (5) for direct comparison

$$\nabla_b T^{ab} = \frac{\lambda F_2}{1 + \lambda f_2} (L_m g^{ab} - T^{ab}) \nabla_b R. \quad (49)$$

One can then easily notice that these two equations become identical if and only if

$$s = -4, \quad T = 2L_m, \quad \Omega^2 = 1 + \lambda f_2(R). \quad (50)$$

The first two conditions give us characteristics of the matter fields and the last just pinpoints the appropriate choice of the conformal factor. The only case that we know of that satisfies the first two conditions (considering also the transformation rule (46)) is a perfect fluid with stiff equation of state $P = \rho$, a particular realization of which is a massless scalar field (which admits a perfect fluid representation). Therefore, if matter is only composed of such fluids or scalars, then a conformal frame can be found in which energy is conserved and the corresponding conformal factor is the one in eq. (50).

We stress once more that the fact that this procedure can not be carried out for all matter fields directly implies that energy conservation is not a generic characteristic of the theory. Also, the fact that massless scalar particles follow geodesics on the one hand, or the fact that we can not show geodesic motion for all fields on the other hand, should not mislead us to think that we can make any statement (positive or negative, respectively) about the EP. In fact, we have already shown above that also null dust, high-frequency photons, and perfect fluid volume elements follow geodesics, even though the use of conformal frames could not provide these results.

Since this is actually the lesson to be learned from this exercise, let us comment on it: In the perfect fluid (including null dust) case, we have already seen that energy is not conserved. Geodesic motion comes as a consequence of the fact that the energy flow is always timelike and parallel to the fluid worldlines instead of being purely spatial. For the case of the electromagnetic field, on the other hand, we already know that the matter action is conformally invariant, so one can not expect that much information can be obtained through the use of conformal frames. The bottom line is that reaching conclusions about geodesic motion and EP violations from energy conservation (the divergence of the stress energy tensor in some conformal frame) is not always an easy and unambiguous task as one may think when considering the metric postulates. This fact further supports the claims made in [21].

3.4. Equivalence principle violations and dark matter

3.4.1. Possible violations of the Equivalence Principle: As already mentioned, the unusual coupling between R and matter in the theories under investigation and the fact that energy is not conserved signal a violation of the EP. On the other hand, in order to get concrete evidence of that, but also in order to derive quantitative results, this issue should be examined further, since there are intricacies in using the metric postulates for judging whether, and how, a theory will show such violations. First of all, let us be more concrete and separate the Strong Equivalence Principle (SEP), the

Einstein Equivalence Principle (EEP), and the Weak Equivalence Principle (WEP) (see [20] for definitions of these concepts).

R couples in the same manner to all possible matter Lagrangians into which one could decide to split L_m (one for each matter field). Therefore, there is no reason to believe that test particles (which by definition do not contribute to the gravitational field) of different compositions will follow different trajectories. This implies that there is no strong indication that the WEP will be violated. However, there is an issue that should be approached with care: to which accuracy can a small particle be considered a test particle in these theories? Could it be that the coupling affects our ability to treat small particles as test particles and neglect their contribution in the gravitational field? This issue, which has received attention within the context of GR [34] needs further investigation.

On the other hand, there is not much to say at this point about the SEP. All theories with more degrees of freedom than GR violate the SEP, and the theory considered here is no exception, even without the R -matter coupling. Obviously, constraints can be imposed by considering the evolution of the effective gravitational coupling. The R -matter coupling should affect gravitational experiments, such as Cavendish experiments. An interesting point is that, since the coupling depends on the curvature, extended bodies whose size is comparable with the radius of curvature can get seriously affected. Considering the equation of motion of extended bodies in such theories will probably provide important constraints.

Finally, one can consider the more accurately tested EEP. It would be interesting to see how the R -matter coupling affects local non-gravitational physics. It should be stressed that this coupling can not be eliminated by a field redefinition, as mentioned above; therefore, going to a local frame can hold surprises. However, it is interesting to note that if $f_2(R)$ is such that $f_2(0) = 0$ then, assuming that spacetime is locally flat in the background by choosing suitable coordinates and treating matter as a perturbation, the effect of the coupling should make its presence felt only at second order and the relevant term should still be suppressed by λ .

All of the above are perspectives for future work. Let us close by discussing how the results derived here relate to that: We have found that massless particles follow null geodesics as usual. This essentially implies that experiments using light, such as redshift experiments, can not constrain the theory. We have also shown that perfect fluid volume elements follow geodesics of the metric $g_{\mu\nu}$. This includes dust, which is usually used as an approximation to study the motion of test particles. This result clashes with the claim that test particles will be affected by the R -coupling, but the issue of how close a small particle is to a test particle remains open. Once more, it should be stressed that geodesic motion alone can not be used to draw conclusions about EP violations. However, one could say that our results hint towards the fact that violations of the EP might be more difficult to detect than expected.

3.4.2. Is there an extra force that can account for dark matter? In [12] the theory under investigation was put forth as a resolution of the dark matter problem in galaxies. It was argued there that the extra force could account for the flat rotational curves of galaxies due to a “MONDian” behaviour. Even though in [12] the extra force was calculated for a perfect fluid, its exact expression was not sufficiently detailed to support the argument of this MOND-like behaviour, which remained rather qualitative. However, our findings cast doubts on whether the theory will really exhibit such a behaviour. As a matter of fact, if we model the galaxy as a perfect fluid (as

the authors of [12] do), then the extra force felt by the stars (the fluid-elements) vanishes and no dark matter phenomenology occurs. Another approximation (and a rough one as well) consists of considering a star in the outskirts of the galaxy as a test particle moving in vacuo under the influence of the gravitational field of the galaxy. In such an approximation, however, any correction to the motion with respect to the one predicted by Newtonian gravity would not be caused by some extra force, as claimed in [12]: it would merely be due to the fact that the vacuum spacetime around the galaxy would differ from the one predicted by GR due to the difference in the field equations.[‡] Therefore, it seems unlikely that an extra force due to the R -matter coupling can actually explain the flat rotation curves of galaxies.

4. Conclusions

We have studied metric $f(R)$ theories of gravity which include an additional direct coupling between the Ricci scalar and the matter Lagrangian. We have shown that they can not be cast into the form of either a usual scalar-tensor theory, *i.e.* one scalar field non-minimally coupled to gravity and minimally coupled to matter, or even a multi-scalar-tensor theory with unusual couplings between the scalar and the matter, such as those studied in [27, 26]. Therefore, they can not easily be dismissed as being equivalent to a theory with known phenomenology. On the contrary, we argued that the R -matter coupling persists (at least implicitly as a coupling between R and a non-dynamical auxiliary field algebraically related to the matter) even after field redefinitions and can not be eliminated by the use of conformal transformations. Hence, the implication of its presence should be thoroughly examined.

We took a first step towards such a study by examining possible deviations of the free fall trajectories from geodesics (of some conformal metric). Massless fields were considered, including a massless scalar field and the electromagnetic field, but in both cases motion followed null geodesics. The same result was obtained for null dust. Remarkably, also for a perfect fluid or a massive scalar, the extra force vanishes and the motion remains geodesic as well. It is worth noting that this does not come as a consequence of energy conservation, but rather as a consequence of the fact that the energy flow is purely timelike and is aligned with the motion.

These results seem to indicate that some of the simple tests of the EP might not be able to discriminate between the theory under consideration and theories that satisfy the EP. Phrased otherwise, this means that this theory might have better chances of escaping the tight constraints related with the EP than other EP-violating theories. However, the same results seem to cast doubts on whether this theory can actually achieve the goal for which it was initially put forth in Ref. [12], *i.e.*, accounting for the observed galactic dynamics without resorting to dark matter.

Although the last argument appears to discourage further study, our overall conclusions do motivate future work on the viability of theories with R -matter coupling, since, besides the fact that they have already been used to address other problems [14, 7, 15, 16] as well, they also appear to be interesting toy models which can help us understanding issues related to the foundations of gravitation theory such as the EP. Additionally, the results presented here are, to some extent, preliminary:

[‡] There have been attempts to address the dark matter problem in this way even in standard $f(R)$ gravity without the R -matter coupling (see for example [35]). Even though in vacuo the R -matter coupling vanishes, the vacuum solutions of the theory considered here and those of $f(R)$ gravity may differ due to the matching with an interior solution.

further work is needed to completely clarify the motion of massive and test particles in the theory, as well as its implications for Local Lorentz Invariance and Local Position Invariance. One of the main steps in this direction is the study of the post-Newtonian limit. Future work can also include stability issues, the Cauchy problem, as well as further investigations of the usefulness of the theory as a substitute for dark energy and dark matter in cosmology and astrophysics.

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Appendix

In this appendix we re-examine a recent claim by one of us in Ref. [28] that the action (2) could be cast into the form of a scalar-tensor theory^{††} by starting from the action

$$S^* = \int d^4x \sqrt{-g} \left\{ \frac{f_1(\phi)}{2} + \frac{1}{2}f'_1(R - \phi) + [1 + \lambda f_2(\phi)]L_m \right\}, \quad (51)$$

instead of (6), and attempting to show that it is also dynamically equivalent to (2). When (51) is varied with respect to ϕ , one obtains

$$\frac{1}{2}(R - \phi)f'_1 + \lambda f'_2 L_m = 0. \quad (52)$$

In the $\lambda = 0$ case (or if matter is absent), and provided that $f'_1 \neq 0$, this yields $R = \phi$ which straightforwardly implies that for this specific choice for λ (corresponding to the usual metric $f(R)$ gravity), the action (51) becomes dynamically equivalent to the reduced version of (2), essentially the action (1). However, it was claimed in [28] that this equivalence continues to hold when $\lambda \neq 0$ (and matter is present). Clearly this can not be true, unless $f'_2 = 0$, in which case the R -matter coupling ceases to exist. Indeed, in order for the action (51) to be dynamically equivalent to (2), eq. (52) should reduce to $R = \phi$. This can not be achieved unless the second term is somehow proportional to $R - \phi$ and the multiplying factor is strictly non-vanishing: this would require at best some unacceptable constraint on the matter Lagrangian, if possible at all. One can therefore conclude that the action (51) can not be dynamically equivalent to (2), contrary to what is claimed in [28].

As a final note on field redefinitions with the use of an auxiliary scalar, let us just mention that, instead of starting from the action (6), one could decide to begin from

$$S'' = \int d^4x \sqrt{-g} \left\{ \frac{f_1(\phi)}{2} + [1 + \lambda f_2(R)]L_m + \Psi(R - \phi) \right\}. \quad (53)$$

Varying this action with respect to Ψ yields $R = \phi$ and one recovers the action (2) as before. However, in this case, variation with respect to ϕ yields

$$\Psi = \frac{1}{2}f'_1 \quad (54)$$

^{††}However, this claim was not actually used in order to obtain the main results of [28].

Assuming that $f_1'' \neq 0$ and replacing eq. (7) back in eq. (53) yields

$$S' = \int d^4x \sqrt{-g} \left\{ \frac{f_1(\phi)}{2} + [1 + \lambda f_2(R)] L_m + \frac{1}{2} f_1'(R - \phi) \right\}. \quad (55)$$

This, unlike the action (51), is dynamically equivalent to the action (2). Using the field redefinition $\Phi = f_1'(\phi)$ and introducing the potential $V(\Phi) = (\phi f_1' - f_1)/2$, the action (55) can take the form

$$S' = \int d^4x \sqrt{-g} \left\{ \frac{\Phi R}{2} - V(\Phi) + [1 + \lambda f_2(R)] L_m \right\}, \quad (56)$$

which is a Brans-Dicke theory with $\omega_0 = 0$ (just like ordinary metric $f(R)$ gravity) but with the addition of the unusual R -matter coupling. This way one can avoid having a non-linear gravitational Lagrangian at the price of introducing a scalar field without touching the R -matter coupling in any way.

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